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Information and Computation 203 (2005) 145–162

Information and Computation

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On the computational power of probabilistic and quantum branching program

Farid Ablayev [a,1], Aida Gainutdinova [b,2], Marek Karpinski [c,3], Cristopher Moore [d,*,4], Christopher Pollett [e]

[a] *Department of Theoretical Cybernetics, Kazan State University, Russia*
[b] *Department of Theoretical Cybernetics, Kazan State University, Russia*
[c] *Department of Computer Science, University of Bonn, Germany*
[d] *Computer Science Department, University of New Mexico, Albuquerque and the Santa Fe Institute, USA*
[e] *Department of Computer Science, San Jose State University, USA*

Received 14 February 2004; revised 22 April 2004
Available online 26 September 2005

Abstract

In this paper, we show that one-qubit polynomial time computations are as powerful as NC^1 circuits. More generally, we define syntactic models for quantum and stochastic branching programs of bounded width and prove upper and lower bounds on their power. We show that any NC^1 language can be accepted exactly by a width-2 quantum branching program of polynomial length, in contrast to the classical case where width 5 is necessary unless $NC^1 = ACC$. This separates width-2 quantum programs from width-2 doubly stochastic programs as we show the latter cannot compute the middle bit of multiplication. Finally, we show that bounded-width quantum and stochastic programs can be simulated by classical programs of larger but bounded width, and thus are in NC^1. For read-oncequantum branching programs (QBPs), we give a symmetric Boolean

* Corresponding author. Fax: +1 505 982 0565.
*E-mail:*ablayev@ksu.ru (F.Ablayev), aida@ksu.ru (A. Gainutdinova), marek@cs.uni-bonn.de (M. Karpinski), moore@cs.unm.edu (C. Moore), pollett@cs.sjsu.edu (C. Pollet).
1 Work done in part while visiting Institute of Advanced Study and Max-Planck Institute for Mathematics, supported in part by Russia Fund for Basic Research Grant 03-01-00769.
2 Supported in part by Russia Fund for Basic Research Grant 03-01-00769.
3 Supported in part by DFG grants, and IST Grant 14036 (RAND-APX).
4 Supported by NSF Grants PHY-0200909, CCR-0220070, and EIA-0218563.

0890-5401/$ - see front matter © 2005 Elsevier Inc. All rights reserved.
doi:10.1016/j.ic.2005.04.003